

- N.B: - Calculators are allowed.
- No credit is awarded for an answer with no justification.

Part A

In this part, for each exercise, indicate whether each statement is true or false and justify your answer.

Exercise 1 (3.5 pts)

We choose a real number x and add 10 to it; then we multiply the result by 3. We deduct 10 from the last result and we divide by 2 the final result. We obtain the number y .

- A) If $x = 0$, then $y = 10$.
- B) If x is an integer, then y is also an integer.
- C) $y = \frac{3}{2}x + 10$
- D) It is impossible to obtain a value of 0 for y .
- E) There is one and only one value of x , such that x is equal to y .

Exercise 2 (2.5 pts)

H is the set of real numbers strictly included between 1,234 and 1,2999.....

- A) 1.2345 belongs to H .
- B) 1.29 belongs to H .
- C) $1 + \frac{1}{3}$ belongs to H .
- D) $1 + \frac{1}{4}$ belongs to H .
- E) $2 - \frac{1}{2}$ belongs to H .

Exercise 3 (5 pts)

During elections, there are 3960 voters who voted for one of the candidates A, B, C. The results are represented by a pie chart. The central angles of the circular sectors are 90° for the votes obtained by candidate A, and 137° for the votes obtained by candidate B

- A) C won.
- B) A obtained one fourth of the votes.
- C) There is less than 1% difference between the votes for B and C
- D) A obtained 25 votes.

E) B had 32 votes more than C.

Exercise 4 (3 pts)

- A) The sum of two even numbers is even.
- B) The sum of two odd numbers is odd.
- C) The product of two even numbers is even.
- D) The product of two odd numbers is odd.
- E) If n is an integer, then n^2+n is odd.

Exercise 5 (3 pts)

Given that $x < 0$

- A) $|-x| = x$
- B) $\sqrt{x^2} = x$
- C) $\sqrt{(x-3)^2} = x-3$
- D) $|3-x| = 3-x$
- E) $|x-2| = x+2$

Part B

In this part, for each exercise, justify your answer.

Exercise 1 (2 pts)

We pick at random a card from a deck of 52 cards. Find the probability of drawing neither an ace nor a club?

Exercise 2 (5 pts)

Consider the numbers: $x = \frac{5}{6}$ and $y = \frac{4}{9}$

- A) Compare x and y
- B) Calculate the sum of the inverses of x and of y .
- C) Calculate the inverse of the sum of x and y .
- D) Calculate the product of the opposites of x and of y .
- E) Find the missing factor $x = \dots \times y$

Exercise 3 (5 pts)

The quarterly average of a student is calculated from 5 homeworks, each graded out of 20. Alan has a average of 14.5 on 4 homeworks.

- A) Could his grades be 13, 14, 15 and 16 ?
- B) Could this average be obtained with one of the grades being 0?
- C) If Alan gets a 17 on his fifth homework, what would his quarterly average be?

- D) Alan claims: «Whatever my score would be on the fifth homework, my average would at least be equal to 12 ». Is he right?

Exercise 4 (2.5 pts)

Calculate $e^{\frac{1}{2}\ln 4} + e^{-\ln \frac{1}{2}}$

Exercise 5 (2.5 pts)

Calculate the derivative of $f(x) = \ln(x^2 + 1) - x$, for any real x .

Exercise 6 (5 pts)

On a particular day, 2650 people visited the Louvre museum. The entrance fee is 10€ per adult and half price per child. The net income for that day was 20500€. How many adults and children visited the Louvre on that day?

Exercise 7 (7 pts)

ABCD is a rhombus with triangle ABD being equilateral. I is the midpoint of [AB], J is the midpoint of [BC], L is the midpoint of [DA], K is the midpoint of [DC].

- A) Show that the triangle LBK is equilateral.
B) What is the nature of the quadrilateral IJKL ? Justify.

Exercise 8 (4 pts)

Alice wants to find the height ST of a tree. She stands at 25 meters from the foot of the tree on a level ground, her eye O being 1.60 m above ground. Her brother sticks a 2.5 m tall pole in the ground at 3.5 meters away from Alice, such that her eye O, the tip A of the pole, and the top S of the tree are aligned. She draws the figure below where (ST) and (AB) are parallel. Calculate the height of the tree.

